Francis turbine part load resonance risk analytical assessment

C. Nicolet, C. Landry, S. Alligné, A. Béguin

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• Context and key goals
• Eigenvalue computation
  ✓ Numerical equations
  ✓ Analytical equations
• Examples of application
  ✓ Hydraulic system 1
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  ✓ Hydraulic system 3
• Conclusions / Take away message
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• Conclusions / Take away message
Context and Key goals

• Massive penetration of alternative renewable energies (Solar, wind power)
• Stochastic nature of the renewable energy production

• To maintain balanced production:
  ✓ Sufficient reserve capacity
  ✓ Primary and secondary control capabilities

→ Hydropower plants
→ Off-design operation

Mean annual growth rates of electricity production 2002-2012
Context and Key goals

- **Frequent power transients:**
  - ✓ Require a wide operating range
  - ✓ May induce high levels of vibration and large pressure fluctuations

Part load condition

Full load condition

* Favrel et al., 2013
* Müller et al., 2013

\[0.2 - 0.4\] • Turbine rotational speed

![Graph](image)

- Pressure fluctuations in the draft tube
- Pressure fluctuations in the spiral case

Resonance excited by the part load vortex

Self-excited instability at full load

HYPERBOLE (ERC/FP7-ENERGY-2013-1-Grant 608532)
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• Conclusions / Take away message
SIMSEN software

- SIMSEN software
  - Hydraulic circuit
  - Electrical installations
  - Rotating inertias
  - Control system

Modeling from water to wire
Electrical Analogy

- Mass and momentum equations:
  \[
  \frac{\partial H}{\partial t} + \frac{d^2}{gA} \frac{\partial Q}{\partial x} = 0
  \]
  \[
  \frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{\lambda |Q|}{2 g DA^2} Q = 0
  \]

- Electrical analogy:
  ✓ (Bergeron, 1950; Paynter, 1953)

- Assumptions:
  ✓ Uniform flow
  ✓ 1-D approach
  ✓ Convective terms neglected
  ✓ Vertical displacements neglected
Eigenvalue computation

- Set of differential equations

\[ [A] \cdot \frac{d\bar{X}}{dt} + [B(\bar{X})] \cdot \bar{X} = C(\bar{x}) \]

- Small perturbation

\[ \bar{X} = \bar{X}_0 + \delta \bar{X} \]

\[ \frac{d(\bar{X}_0 + \delta \bar{X})}{dt} = f(\bar{X}_0 + \delta \bar{X}) \]

- Eigenfrequency

\[ \det \left( [I] \cdot s + [A_l]^{-1} [B_l] \right) = 0 \]
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• Context and key goals
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Natural frequencies

- Hydraulic system modelled by an equivalent pipe

\[ l_{tot} = \sum_{i=1}^{n} l_i \]

\[ a_{equ} = \frac{l_{tot}}{\sum_{i=1}^{n} \frac{l_i}{a_i}} \]

\[ f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \]

Total length
Equivalent wave speed
Natural frequency (Order k)

Standing wave
First natural frequency of the cavitating draft tube

- Simplified hydroacoustic model of the frictionless cavitating draft tube

\[ C_{DT} = \frac{l_{DT} \cdot g \cdot \bar{A}_{DT}}{a_{DT}^2} \]

\[ L_{DT} = \frac{l_{DT}}{g \cdot \bar{A}_{DT}} \]

\[ f_0 = \frac{1}{2 \cdot \pi} \sqrt{\frac{1}{L_{DT} \cdot C_{DT}}} = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{l_{DT}} \]

\[ f_0 = \frac{1}{2 \pi} \sqrt{\frac{k}{m}} \approx \frac{1}{2 \pi} \sqrt{\frac{1}{c_{DT} l_{DT}}} \]

\[ l_{DT} = \text{DT length [m]} \]
\[ A_{DT} = \text{DT cross section area [m}^2\text{]} \]
\[ a_{DT} = \text{DT wave speed [m/s]} \]
First natural frequency of the cavitating draft tube

- Simplified hydroacoustic model of the frictionless cavitating draft tube

Cavitation compliance

$$C_{DT} = \frac{l_{DT} \cdot g \cdot A_{DT}}{a_{DT}^2}$$

Tailrace pipe inductance

$$L_{TR} = \frac{l_{TR}}{g \cdot A_{TR}}$$

First natural frequency

$$f_o = \frac{1}{2 \cdot \pi} \sqrt{\frac{1}{L_{TR} \cdot C_{DT}}} = \frac{1}{2 \cdot \pi} \sqrt{\frac{a_{DT}}{l_{DT} \cdot l_{TR} \cdot A_{DT} / A_{TR}}}$$

- Draft tube
- Tailrace pipe

\[\begin{align*}
l_{TR} & = \text{TR pipe length [m]} \\
A_{TR} & = \text{TR pipe cross section area [m}^2\text{]} \\
l_{TR} \cdot A_{TR} & = \text{Cavitation compliance} \\
L_{TR} & = \text{Tailrace pipe inductance} \\
f_o & = \text{First natural frequency} \quad \text{Dörfler, 2013}
\end{align*}\]
### Analytical equations

#### Summary

<table>
<thead>
<tr>
<th>Equivalent pipe</th>
<th>Draft tube without TR</th>
<th>Draft tube with TR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1st order</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_1 = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}}$</td>
<td>$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{l_{DT}}$</td>
<td>$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} A_{DT} A_{TR}}}$</td>
</tr>
<tr>
<td><strong>2nd-6th order</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$</td>
<td>$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$</td>
<td>$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$</td>
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Example of applications

• Hydraulic system 1

Penstock
- \( L = 300 \text{ m} \)
- \( D = 1.2 \text{ m} \)
- \( a = 1'250 \text{ m/s} \)
- \( \lambda = 0.012 \text{ -} \)

Turbine
- \( P_n = 5 \text{ MW} \)
- \( Q_n = 5.0 \text{ m}^3/\text{s} \)
- \( H_n = 100 \text{ mWC} \)
- \( N_n = 750 \text{ rpm} \)
- \( D_{ref} = 0.846 \text{ m} \)
- \( N_q = 53 \text{ -} \)

Draft tube
- \( L = 10 \text{ m} \)
- \( D = 1.2 \text{ m} \)
- \( a = [50-100] \text{ m/s} \)
- \( \lambda = 0.012 \text{ -} \)

\( N_n = 750 \text{ rpm} = 12.5 \text{ Hz} \)

\( f_{\text{excitation}} = [0.2 - 0.4] \cdot f_n \)

\( f_{\text{excitation}} = [2.5 - 5 \text{ Hz}] \)
Example of applications

- **Hydraulic system 1**

  **Draft tube**

  **Penstock**

  **Turbine**

  **Elastic mode shape**
  - Non-Linear amplitude variation of pressure in the TR as function of the length
  - Non-constant amplitude variation of discharge in the TR as function of the length

  \[ f_1 = \frac{a_{\text{equ}}}{\lambda_1} = \frac{a_{\text{equ}}}{2 \cdot l_{\text{tot}}} \cdot 1 \]

  \[ f_o = \frac{1}{2 \pi} \frac{a_{DT}}{l_{DT}} \]
Example of applications

- **Hydraulic system 1**

  - 1\textsuperscript{st} order: Better agreement with \( f_1 \)
  - 2\textsuperscript{nd}-6\textsuperscript{th} order: Rather good agreement for natural frequencies
    Maximum error of 14%.
  - Risk of resonance with the draft tube in red.

**System 1**

<table>
<thead>
<tr>
<th>( f_0 ) [Hz]</th>
<th>Analytical calculation a DT (min) [m/s]</th>
<th>Eigen value calculation a DT (min) [m/s]</th>
<th>Relative error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>f0</td>
<td>0.8</td>
<td>1.24</td>
<td>-35.48</td>
</tr>
<tr>
<td>f1</td>
<td>1.14</td>
<td>1.24</td>
<td>-8.00</td>
</tr>
<tr>
<td>f2</td>
<td>2.27</td>
<td>2.09</td>
<td>8.61</td>
</tr>
<tr>
<td>f3</td>
<td>3.41</td>
<td>3.67</td>
<td>-7.08</td>
</tr>
<tr>
<td>f4</td>
<td>4.55</td>
<td>4.18</td>
<td>8.85</td>
</tr>
<tr>
<td>f5</td>
<td>5.68</td>
<td>5.99</td>
<td>-5.18</td>
</tr>
<tr>
<td>f6</td>
<td>6.82</td>
<td>6.15</td>
<td>10.89</td>
</tr>
</tbody>
</table>

\( f_{\text{excitation}} = [2.5 - 5 \text{ Hz}] \)

\( a = 50 \text{ m/s} \)

\( f_1 = \frac{a_{\text{equ}}}{\lambda_1} = \frac{a_{\text{equ}}}{2 \cdot l_{\text{tot}}} \)

\( f_o = \frac{1}{2 \cdot \pi} \frac{a_{\text{DT}}}{l_{\text{DT}}} \)
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Example of applications

- **Hydraulic system 2**

<table>
<thead>
<tr>
<th>Component</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Penstock</strong></td>
<td>L = 300 m, D = 1.2 m, a = 1'250 m/s, ( \lambda = 0.012 )</td>
</tr>
<tr>
<td><strong>Turbine</strong></td>
<td>( P_n = 5 ) MW, ( Q_n = 5.0 ) m³/s, ( H_n = 100 ) mWC, ( N_n = 750 ) rpm, ( D_{ref} = 0.846 ) m, ( N_q = 53 )</td>
</tr>
<tr>
<td><strong>Draft tube</strong></td>
<td>L = 10 m, D = 1.2 m, a = [50-100] m/s, ( \lambda = 0.012 )</td>
</tr>
<tr>
<td><strong>Tailrace pipe</strong></td>
<td>L = 100 m, D = 1.2 m, a = 1'250 m/s, ( \lambda = 0.012 )</td>
</tr>
</tbody>
</table>

- \( N_n = 750 \) rpm = 12.5 Hz

\( f_{excitation} = [0.2 – 0.4] \cdot f_n \)

\( f_{excitation} = [2.5 – 5] \) Hz
Example of applications

- **Hydraulic system 2**

  - **Draft tube**
    - **Penstock**
    - **Turbine**
    - **Tailrace pipe**

  - **Rigid column mode shape**
    - Linear amplitude variation of pressure in the TR
    - Constant amplitude variation of discharge in the TR
    - Similar to surge tank mass oscillation between TR pipe and DT compliance

  - **Equation**
    \[
    f_1 = \frac{a_{\text{equ}}}{2 \cdot \pi \cdot l_{\text{tot}}}
    \]
    \[
    f_o = \frac{1}{2 \cdot \pi} \frac{a_{\text{DT}}}{\sqrt{l_{\text{DT}} \cdot A_{\text{DT}} / A_{\text{TR}}}}
    \]

  - **Modes**
    - **1st Pressure mode**
    - **1st Discharge mode**
    - **Rigid column mode shape**
Example of applications

- **Hydraulic system 2**

  \[ f_{\text{excitation}} = [2.5 \text{ – } 5 \text{ Hz}] \]

<table>
<thead>
<tr>
<th>( f_0 \text{ [Hz]} )</th>
<th>( f_1 \text{ [Hz]} )</th>
<th>( f_2 \text{ [Hz]} )</th>
<th>( f_3 \text{ [Hz]} )</th>
<th>( f_4 \text{ [Hz]} )</th>
<th>( f_5 \text{ [Hz]} )</th>
<th>( f_6 \text{ [Hz]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical calculation ( a \text{ DT (min)} ) [m/s]</td>
<td>Eigen value calculation ( a \text{ DT (min)} ) [m/s]</td>
<td>Relative error [%]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.27</td>
<td>-7.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.96</td>
<td>0.27</td>
<td>255.56</td>
<td></td>
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</tr>
<tr>
<td>1.92</td>
<td>2.04</td>
<td>-5.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.88</td>
<td>2.53</td>
<td>13.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.85</td>
<td>4.12</td>
<td>-6.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.81</td>
<td>4.87</td>
<td>-1.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.77</td>
<td>5.16</td>
<td>11.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( f_0 \text{ [Hz]} )</th>
<th>( f_1 \text{ [Hz]} )</th>
<th>( f_2 \text{ [Hz]} )</th>
<th>( f_3 \text{ [Hz]} )</th>
<th>( f_4 \text{ [Hz]} )</th>
<th>( f_5 \text{ [Hz]} )</th>
<th>( f_6 \text{ [Hz]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical calculation ( a \text{ DT (max)} ) [m/s]</td>
<td>Eigen value calculation ( a \text{ DT (max)} ) [m/s]</td>
<td>Relative error [%]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.55</td>
<td>-9.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.19</td>
<td>0.55</td>
<td>116.36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.38</td>
<td>2.1</td>
<td>13.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.57</td>
<td>4.05</td>
<td>-11.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.76</td>
<td>4.88</td>
<td>-2.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.95</td>
<td>6.18</td>
<td>-3.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.14</td>
<td>6.36</td>
<td>12.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **1**\(^{\text{st}}\) order: Good agreement with \( f_0 \)
- **2**\(^{\text{nd}}\)–**6**\(^{\text{th}}\) order: Rather good agreement for natural frequencies
  Maximum error of 14%.
- Risk of resonance with the draft tube in red.
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Example of applications

- **Hydraulic system 3**

  - **Penstock**
    - \( L = 300 \text{ m} \)
    - \( D = 1.2 \text{ m} \)
    - \( a = 1'250 \text{ m/s} \)
    - \( \lambda = 0.012 \)  

  - **Turbine**
    - \( P_n = 5 \text{ MW} \)
    - \( Q_n = 5.0 \text{ m}^3/\text{s} \)
    - \( H_n = 100 \text{ mWC} \)
    - \( N_n = 750 \text{ rpm} \)
    - \( D_{ref} = 0.846 \text{ m} \)
    - \( N_q = 53 \)  

  - **Draft tube**
    - \( L = 10 \text{ m} \)
    - \( D = 1.2 \text{ m} \)
    - \( a = [50-100] \text{ m/s} \)
    - \( \lambda = 0.012 \)  

  - **Tailrace pipe**
    - \( L = 100 \text{ m} \)
    - \( D = 2 \text{ m} \)
    - \( a = 1'250 \text{ m/s} \)
    - \( \lambda = 0.012 \)

\[ N_n = 750 \text{ rpm} = 12.5 \text{ Hz} \]
\[ f_{excitation} = [0.2 - 0.4] \cdot f_n \]
\[ f_{excitation} = [2.5 - 5 \text{ Hz}] \]
Example of applications

- **Hydraulic system 3**

  - **Draft tube**
    - 1st Pressure mode
    - Rigid column mode shape
    - 1st Discharge mode

  - **Penstock**
  - **Turbine**
  - **Tailrace pipe**

  - **Rigid column mode shape**
    - Linear amplitude variation of pressure in the TR
    - Constant amplitude variation of discharge in the TR
    - Similar to surge tank mass oscillation between TR pipe and DT compliance

\[
f_1 = \frac{\alpha_{equ}}{A_{TR}} \frac{\alpha_{equ}}{2 \cdot l_{tot}}
\]

\[
f_o = \frac{1}{2 \cdot \pi} \frac{A_{DT} \sqrt{l_{DT} \cdot l_{TR}}}{A_{TR}}
\]
### Example of applications

- **Hydraulic system 3**

![Diagram of a hydraulic system with a draft tube and a turbine connected via a penstock]

- **1\textsuperscript{st} order**: Good agreement with $f_0$
- **2\textsuperscript{nd}–6\textsuperscript{th} order**: Rather good agreement for natural frequencies. Maximum error of 14%.
- **Risk of resonance with the draft tube in red.**

#### Analytical calculation

<table>
<thead>
<tr>
<th>System 3</th>
<th>Analytical calculation</th>
<th>Eigen value calculation</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a DT (max)</td>
<td>a DT (max)</td>
<td>[%]</td>
</tr>
<tr>
<td></td>
<td>[m/s]</td>
<td>[m/s]</td>
<td></td>
</tr>
<tr>
<td>$f_0$ [Hz]</td>
<td>0.84</td>
<td>0.81</td>
<td>3.70</td>
</tr>
<tr>
<td>$f_1$ [Hz]</td>
<td>1.19</td>
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<td>46.91</td>
</tr>
<tr>
<td>$f_2$ [Hz]</td>
<td>2.38</td>
<td>2.1</td>
<td>13.33</td>
</tr>
<tr>
<td>$f_3$ [Hz]</td>
<td>3.57</td>
<td>4.03</td>
<td>-11.41</td>
</tr>
<tr>
<td>$f_4$ [Hz]</td>
<td>4.76</td>
<td>4.72</td>
<td>0.85</td>
</tr>
<tr>
<td>$f_5$ [Hz]</td>
<td>5.95</td>
<td>6.14</td>
<td>-3.09</td>
</tr>
<tr>
<td>$f_6$ [Hz]</td>
<td>7.14</td>
<td>6.55</td>
<td>9.01</td>
</tr>
</tbody>
</table>

#### Analytical calculation

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<th>Relative error</th>
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<tr>
<td></td>
<td>a DT (min)</td>
<td>a DT (min)</td>
<td>[%]</td>
</tr>
<tr>
<td></td>
<td>[m/s]</td>
<td>[m/s]</td>
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<tr>
<td>$f_0$ [Hz]</td>
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<tr>
<td>$f_1$ [Hz]</td>
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<td>$f_2$ [Hz]</td>
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<td>2.04</td>
<td>-5.88</td>
</tr>
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<td>$f_3$ [Hz]</td>
<td>2.88</td>
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<td>12.94</td>
</tr>
<tr>
<td>$f_4$ [Hz]</td>
<td>3.85</td>
<td>4.12</td>
<td>-6.55</td>
</tr>
<tr>
<td>$f_5$ [Hz]</td>
<td>4.81</td>
<td>4.84</td>
<td>-0.62</td>
</tr>
<tr>
<td>$f_6$ [Hz]</td>
<td>5.77</td>
<td>6.16</td>
<td>-6.33</td>
</tr>
</tbody>
</table>

**Excitation $f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$**

- **Risk of resonance with the draft tube in red.**

- **Hydraulic system 3**

- **Excitation** $f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$
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Conclusions

• Analytical approach to determine the possible risk of resonance with the draft tube vortex rope excitation (2.5-5Hz)

• 1\textsuperscript{st} order: Better agreement with $f_1$ for the hydraulic system without a tailrace pipe (hydraulic system 1) → Elastic mode shape
• 1\textsuperscript{st} order: Good agreement with $f_0$ for the hydraulic system with a tailrace pipe (hydraulic system 2 & 3) → rigid column mode shape

• 2\textsuperscript{nd}-6\textsuperscript{th} order: Rather good agreement for natural frequencies (Maximum error of 14%).

• Limitations of the methodology
  ✓ Parallel branches:
    • Modelling by a single branch with equivalent parameters to obtain a first order of magnitude.
    • Real system will feature much more complex and numerous eigenvalues (hydraulic system asymmetry, diameters, bifurcations)
Take away Message

- This analytical method is included as ANNEXE E.2 of the new IEC Technical Specification 62882 ED1 (to be issued in 2020)

Without a tailrace pipe

1st order

\[ f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k \]

2nd–6th order

\[ f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k \]

With a tailrace pipe

1st order

\[ f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \cdot A_{DT} \cdot A_{TR}}} \]

2nd–6th order

\[ f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k \]

Hydraulic machines – IEC Technical specification for Francis turbine pressure fluctuation
Thank you for your attention!
Example of applications

<table>
<thead>
<tr>
<th></th>
<th>Hydraulic system 1</th>
<th>Hydraulic system 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>$f_2$</td>
<td><img src="image3" alt="Diagram" /></td>
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<tr>
<td>$f_3$</td>
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