



# Francis turbine part load resonance risk analytical assessment

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**SIMSEN**

# Contents

- Context and key goals
- Eigenvalue computation
  - ✓ Numerical equations
  - ✓ Analytical equations
- Examples of application
  - ✓ Hydraulic system 1
  - ✓ Hydraulic system 2
  - ✓ Hydraulic system 3
- Conclusions / Take away message

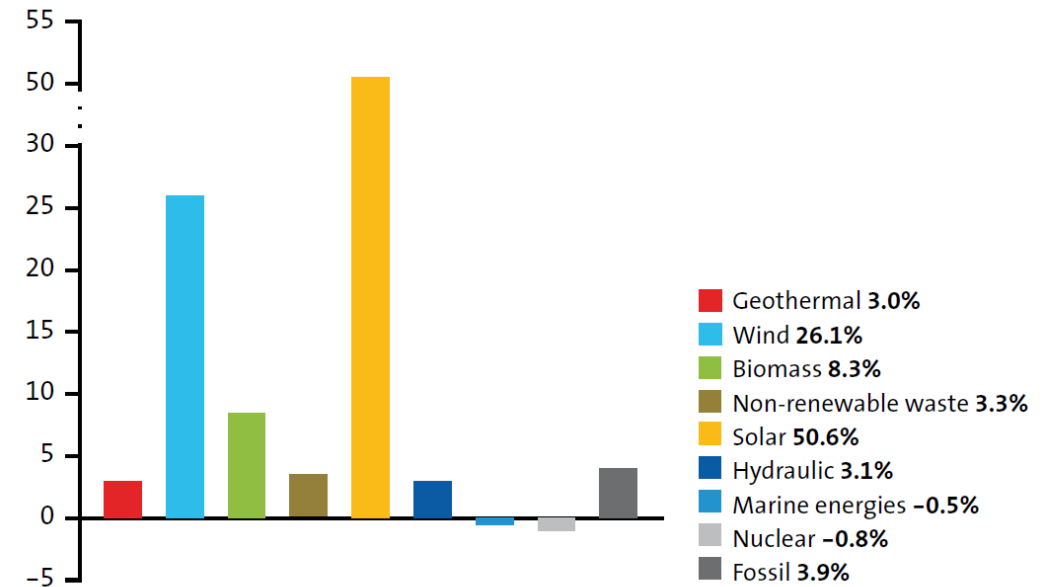
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## Context and Key goals

- Massive penetration of alternative renewable energies (Solar, wind power)
- Stochastic nature of the renewable energy production

- To maintain balanced production:
  - ✓ Sufficient reserve capacity
  - ✓ Primary and secondary control capabilities
- Hydropower plants
- Off-design operation

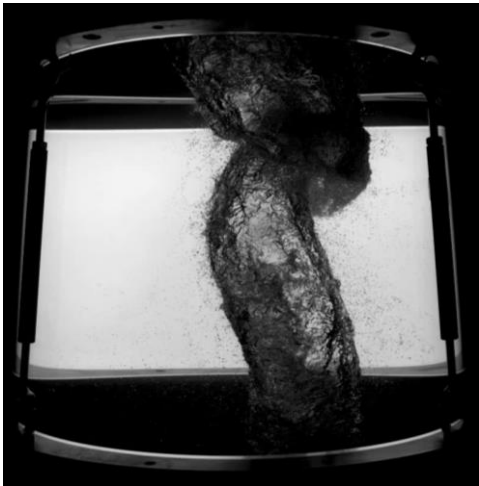


Mean annual growth rates of electricity  
production 2002-2012

# Context and Key goals

- Frequent power transients:
  - ✓ Require a wide operating range
  - ✓ May induce high levels of vibration and large pressure fluctuations

Part load condition



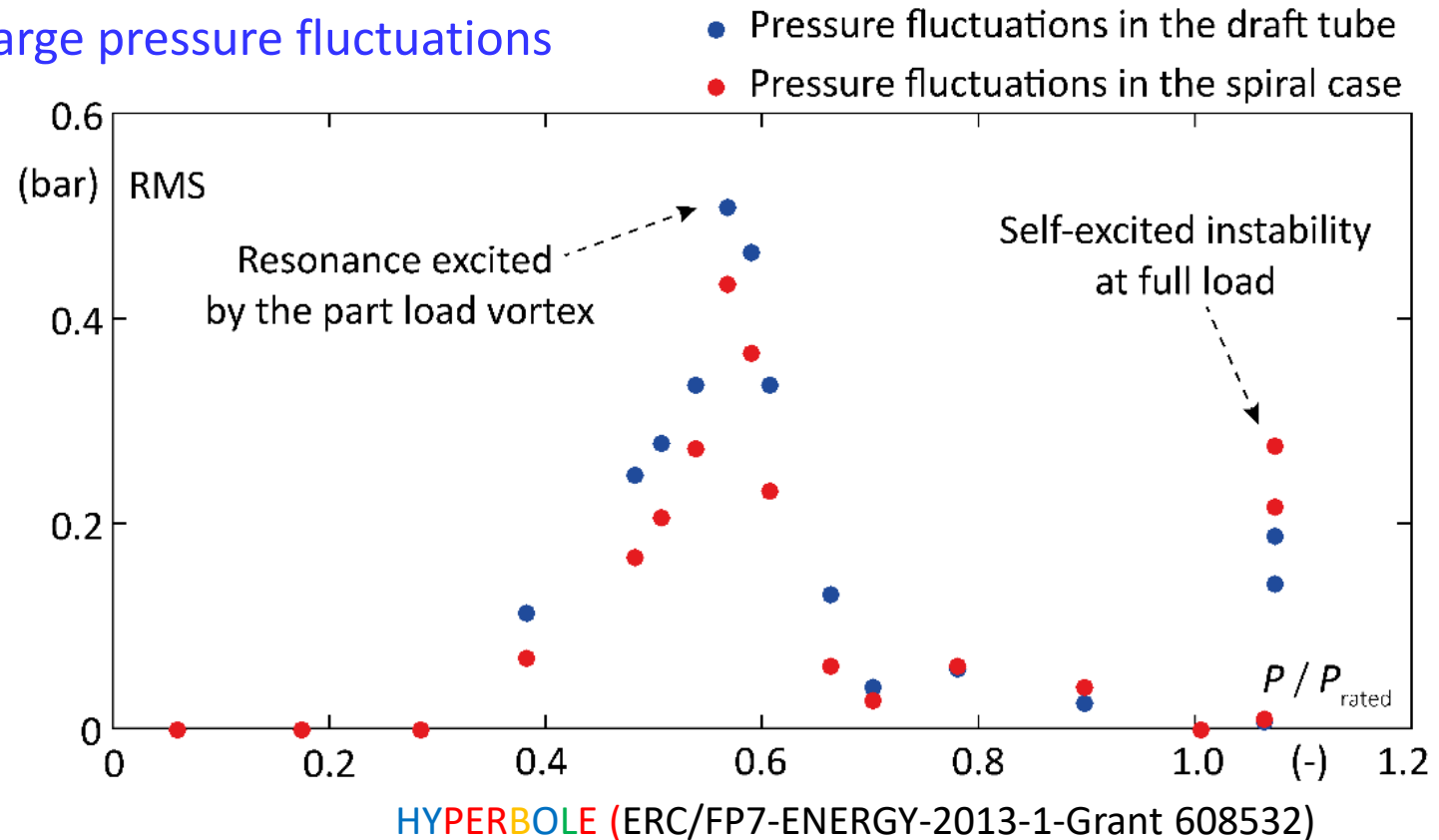
\* Favrel *et al.*, 2013

Full load condition



\* Müller *et al.*, 2013

[0.2 – 0.4] • Turbine rotational speed



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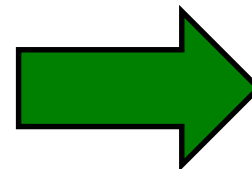
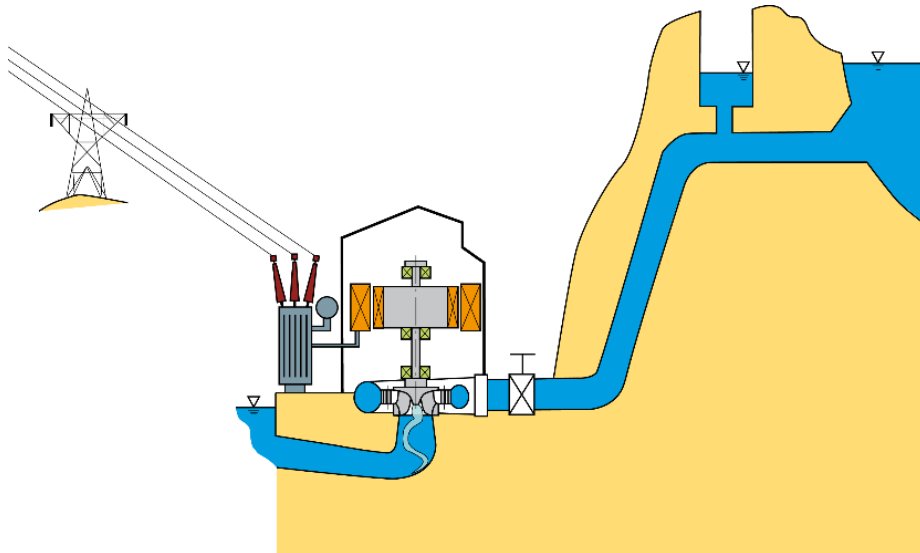
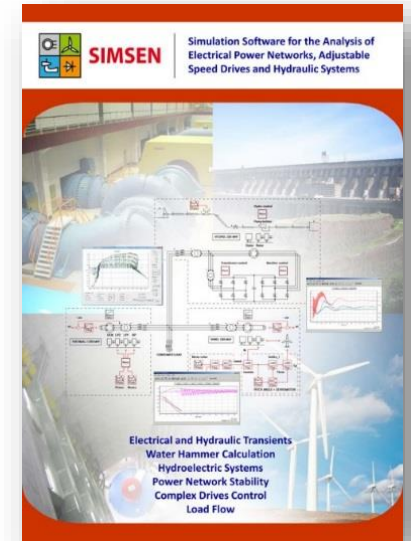
# SIMSEN software

- SIMSEN software
  - ✓ Hydraulic circuit
  - ✓ Electrical installations
  - ✓ Rotating inertias
  - ✓ Control system

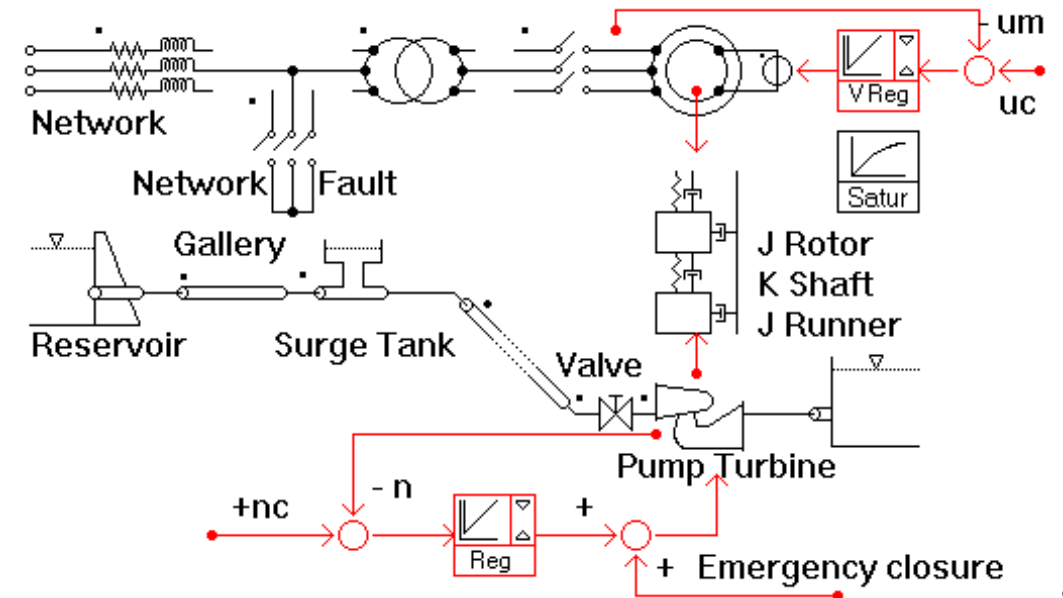


**SIMSEN**

**EPFL**

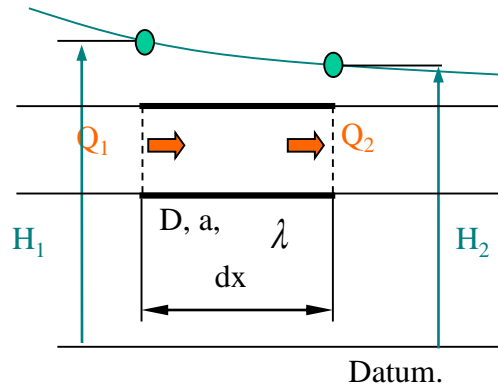


**Modeling  
from  
water  
to wire**



# Electrical Analogy

- Mass and momentum equations:



$$\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0$$

**Storage**

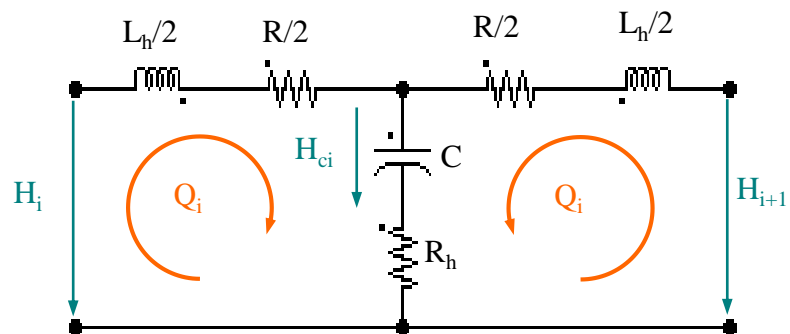
$$\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{\lambda |Q|}{2gDA^2} Q = 0$$

**Losses**

**Inertia**

- Electrical analogy:

✓ (Bergeron, 1950; Paynter, 1953)



$$\frac{\partial U}{\partial t} + \frac{1}{C_e} \frac{\partial I}{\partial x} = 0$$

$$\frac{\partial U}{\partial x} + \frac{1}{L_e} \frac{\partial I}{\partial t} + R_e I = 0$$

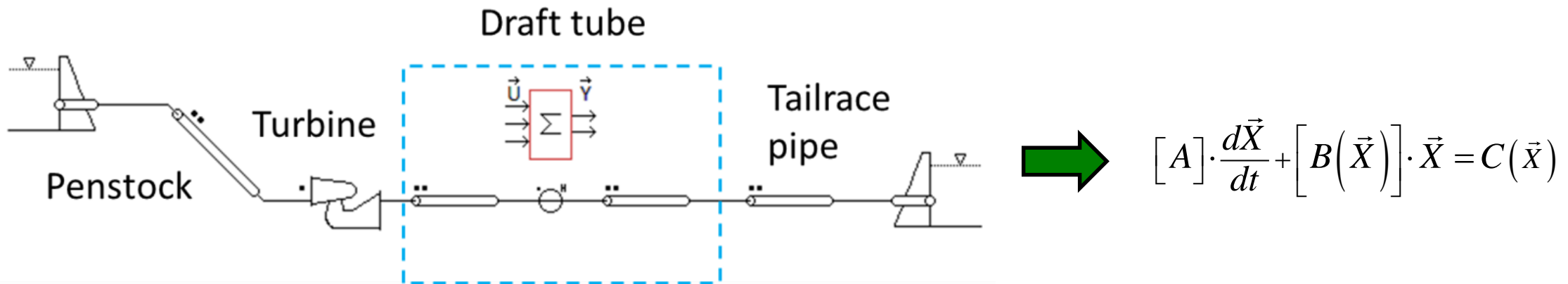
- Assumptions:

- ✓ Uniform flow
- ✓ 1-D approach
- ✓ Convective terms neglected
- ✓ Vertical displacements neglected



# Eigenvalue computation

- Set of differential equations



- Small perturbation

- Eigenfrequency

$$\vec{X} = \vec{X}_0 + \delta \vec{X} \quad \left\{ \begin{array}{l} \frac{d(\vec{X}_0 + \delta \vec{X})}{dt} = \vec{f}(\vec{X}_0 + \delta \vec{X}) \end{array} \right. \quad \Rightarrow \quad \det \left( [I] \cdot s + [A_l]^{-1} [B_l] \right) = 0$$

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# Natural frequencies

- Hydraulic system modelled by an equivalent pipe

Total length

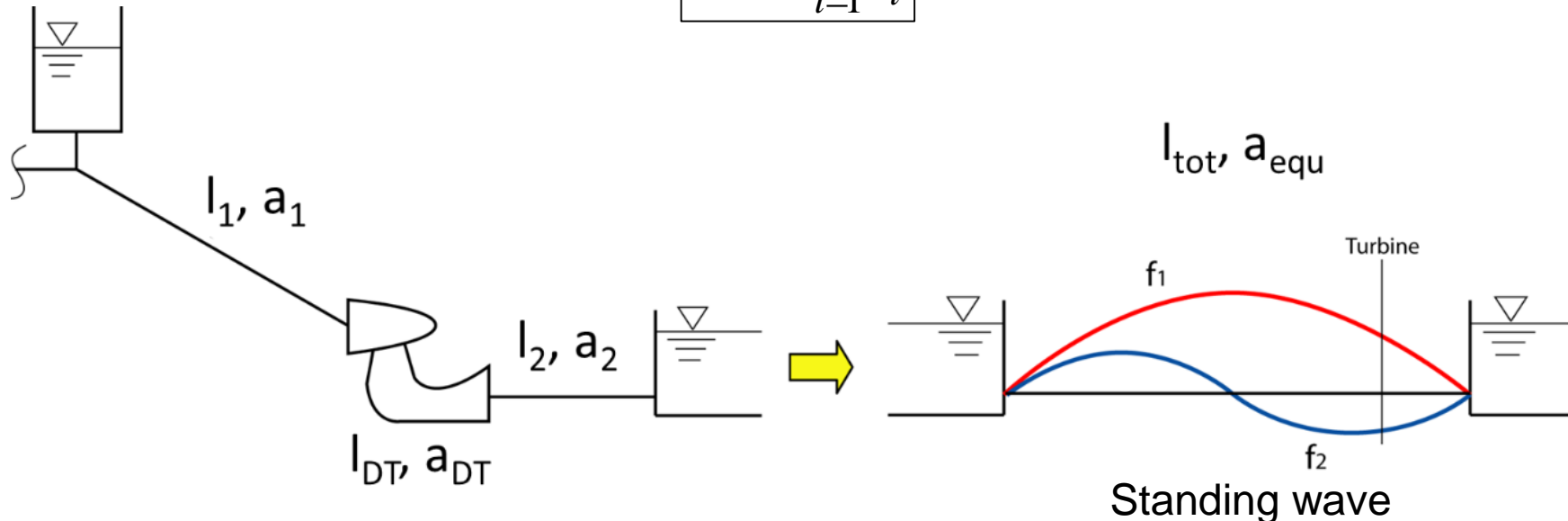
$$l_{tot} = \sum_{i=1}^n l_i$$

Equivalent wave speed

$$a_{equ} = \frac{l_{tot}}{\sum_{i=1}^n \frac{l_i}{a_i}}$$

Natural frequency (Order k)

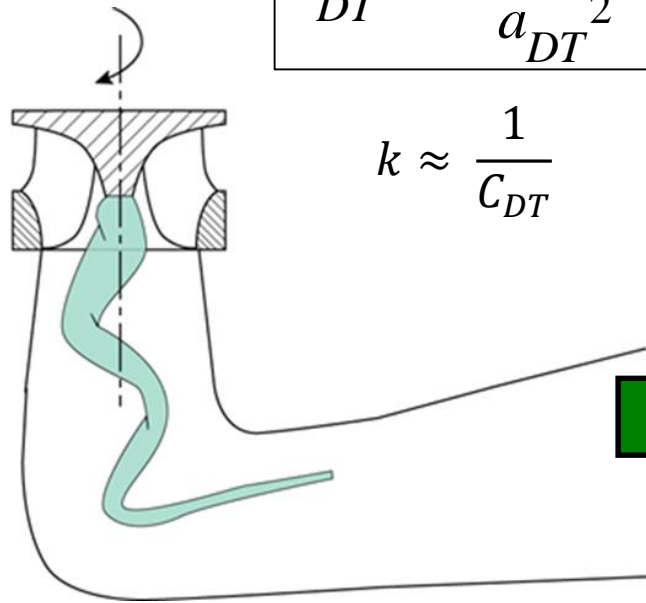
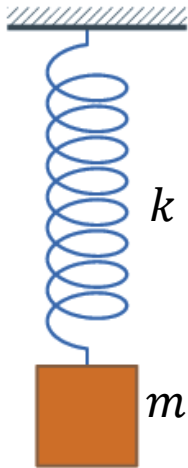
$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$$



# First natural frequency of the cavitating draft tube

- Simplified hydroacoustic model of the frictionless cavitating draft tube

Similar to a spring-mass system



Draft tube

Cavitation compliance

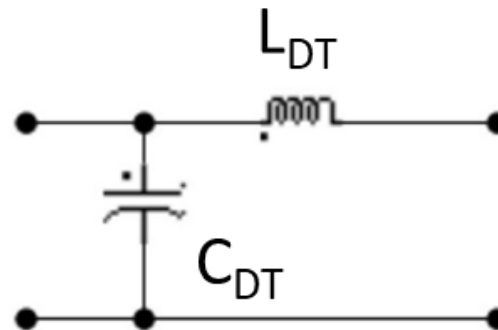
$$C_{DT} = \frac{l_{DT} \cdot g \cdot \bar{A}_{DT}}{a_{DT}^2}$$

$$k \approx \frac{1}{C_{DT}}$$

Draft tube inductance

$$L_{DT} = \frac{l_{DT}}{g \cdot A_{DT}}$$

$$m \approx L_{DT}$$



First natural frequency \* Jacob, 1993

$$f_o = \frac{1}{2 \cdot \pi} \frac{1}{\sqrt{L_{DT} \cdot C_{DT}}} = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{l_{DT}}$$

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \approx \frac{1}{2\pi} \sqrt{\frac{1}{C_{DT} L_{DT}}}$$

$l_{DT}$	=	DT length [m]
$A_{DT}$	=	DT cross section area [m <sup>2</sup> ]
$a_{DT}$	=	DT wave speed [m/s]

# First natural frequency of the cavitating draft tube

- Simplified hydroacoustic model of the frictionless cavitating draft tube

Cavitation compliance

$$C_{DT} = \frac{l_{DT} \cdot g \cdot \bar{A}_{DT}}{a_{DT}^2}$$

Tailrace pipe inductance

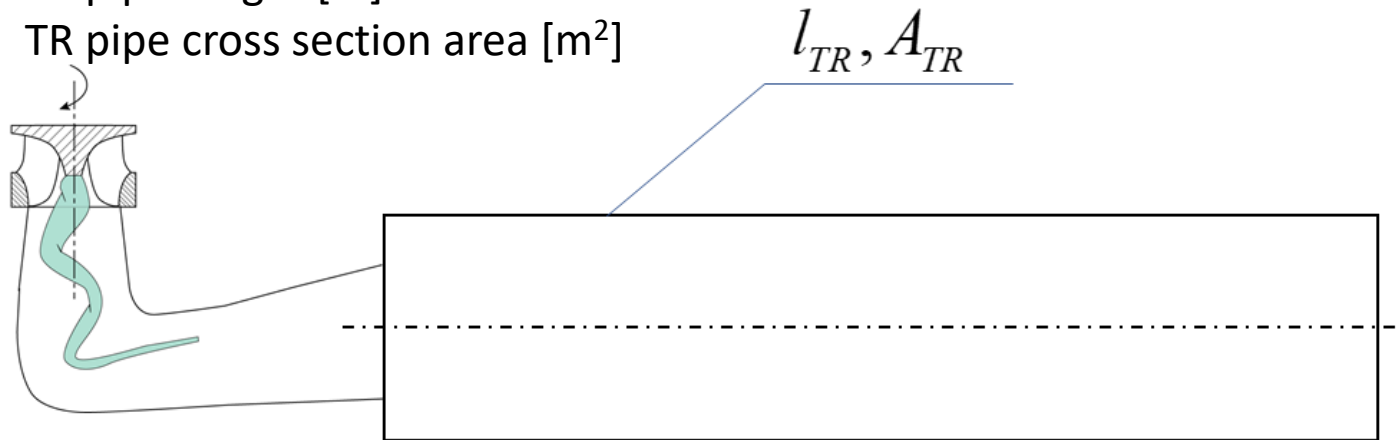
$$L_{TR} = \frac{l_{TR}}{g \cdot A_{TR}}$$

First natural frequency \* Dörfler, 2013

$$f_o = \frac{1}{2 \cdot \pi} \frac{1}{\sqrt{L_{TR} \cdot C_{DT}}} = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \frac{A_{DT}}{A_{TR}}}}$$

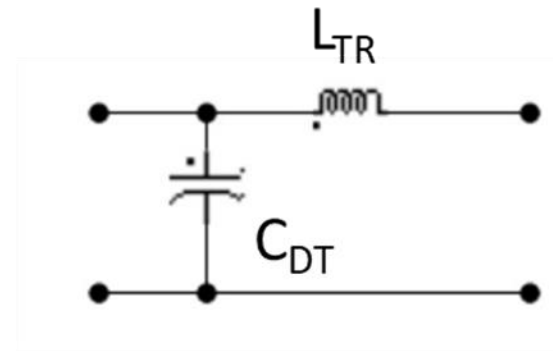
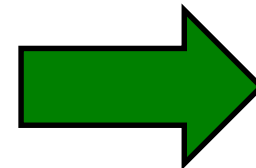
$l_{TR}$  = TR pipe length [m]

$A_{TR}$  = TR pipe cross section area [m<sup>2</sup>]



Draft tube

Tailrace pipe



# Analytical equations

- Summary

**Equivalent  
pipe**

**Draft tube  
without TR**

**Draft tube  
with TR**

**1<sup>st</sup> order**

$$f_1 = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}}$$

$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{l_{DT}}$$

$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \frac{A_{DT}}{A_{TR}}}}$$

**2<sup>nd</sup>-6<sup>th</sup> order**

$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$$

$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$$

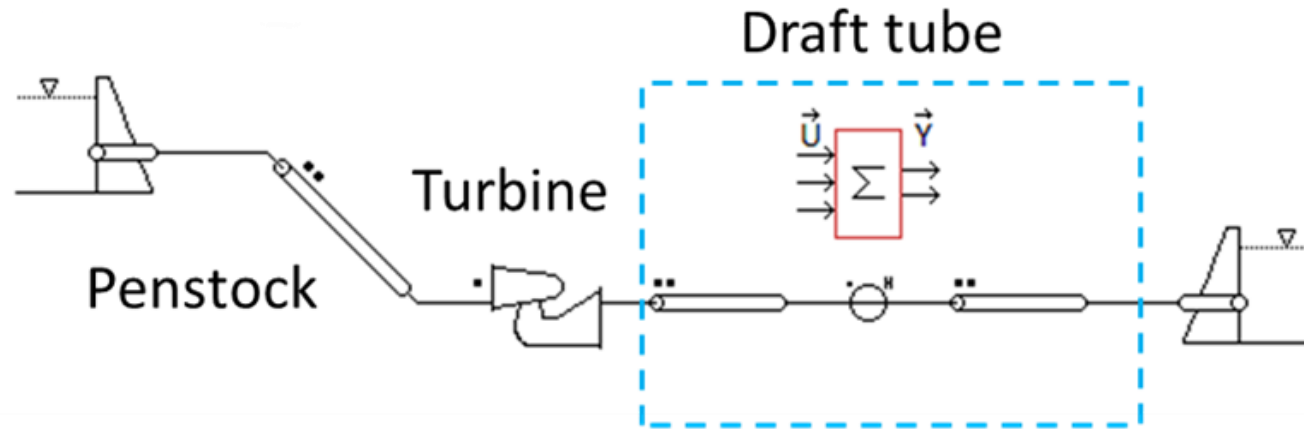
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# Example of applications

- Hydraulic system 1



$$N_n = 750 \text{ rpm} = 12.5 \text{ Hz}$$

$$\rightarrow f_{\text{excitation}} = [0.2 - 0.4] \cdot f_n$$

$$\rightarrow f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$$

Penstock		
L	= 300	m
D	= 1.2	m
a	= 1'250	m/s
λ	= 0.012	-

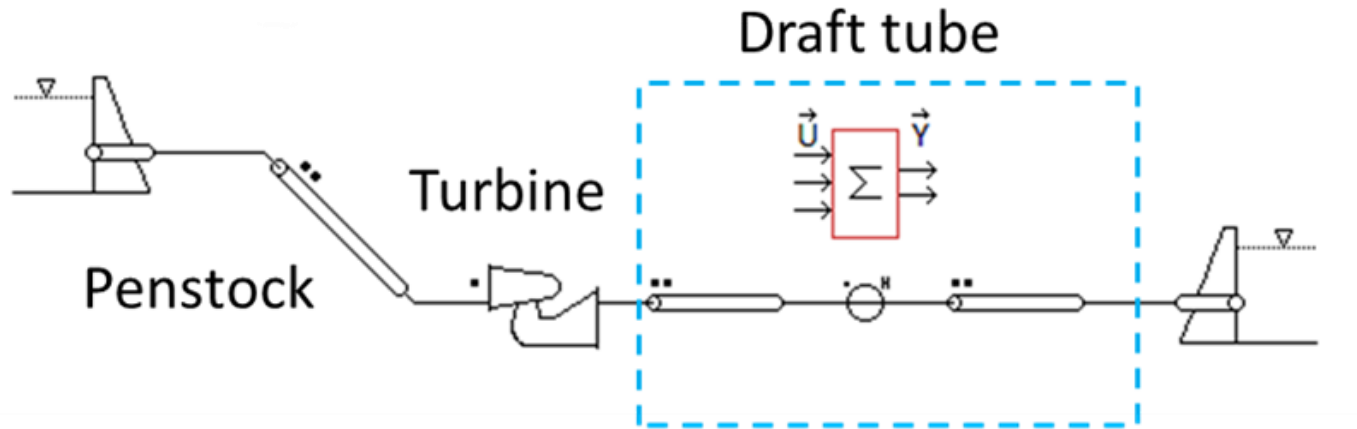
Turbine		
P <sub>n</sub>	= 5	MW
Q <sub>n</sub>	= 5.0	m <sup>3</sup> /s
H <sub>n</sub>	= 100	mWC
N <sub>n</sub>	= 750	rpm
D <sub>ref</sub>	= 0.846	m
N <sub>q</sub>	= 53	-

Draft tube		
L	= 10	m
D	= 1.2	m
a	= [50-100]	m/s
λ	= 0.012	-



# Example of applications

- Hydraulic system 1

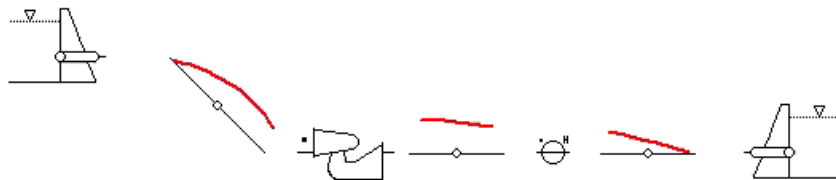


## Elastic mode shape

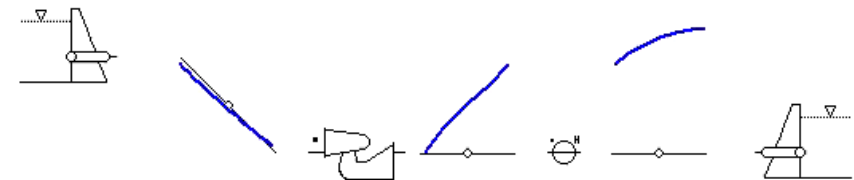
- ✓ Non-Linear amplitude variation of pressure in the TR as function of the length
- ✓ Non-constant amplitude variation of discharge in the TR as function of the length

$$f_1 = \frac{a_{equ}}{\lambda_1} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot 1$$

~~$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{l_{DT}}$$~~



1<sup>st</sup> Pressure mode



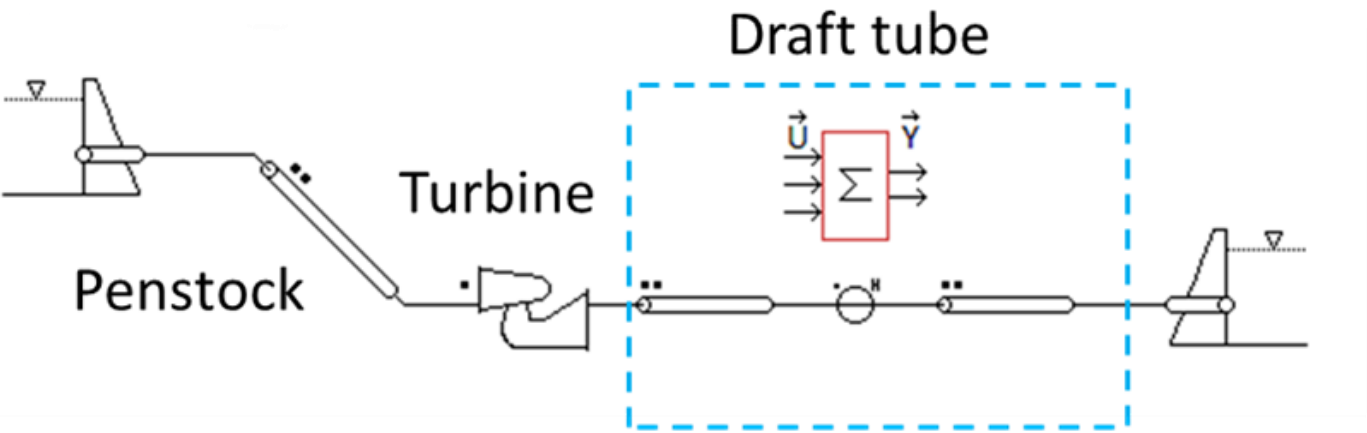
Elastic mode shape

1<sup>st</sup> Discharge mode

# Example of applications

$f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$

- Hydraulic system 1



- 1<sup>st</sup> order: Better agreement with  $f_1$
- 2<sup>nd</sup>-6<sup>th</sup> order: Rather good agreement for natural frequencies  
Maximum error of 14%.
- Risk of resonance with the draft tube in red.

$a = 50 \text{ m/s}$

System 1	Analytical calculation	Eigen value calculation	
	a DT (min)	a DT (min)	Relative error
	[m/s]	[m/s]	[%]
$f_0$ [Hz]	0.8	1.24	-35.48
$f_1$ [Hz]	1.14	1.24	-8.06
$f_2$ [Hz]	2.27	2.09	8.61
$f_3$ [Hz]	3.41	3.67	-7.08
$f_4$ [Hz]	4.55	4.18	8.85
$f_5$ [Hz]	5.68	5.99	-5.18
$f_6$ [Hz]	6.82	6.15	10.89

$$f_1 = \frac{a_{equ}}{\lambda_1} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot 1$$

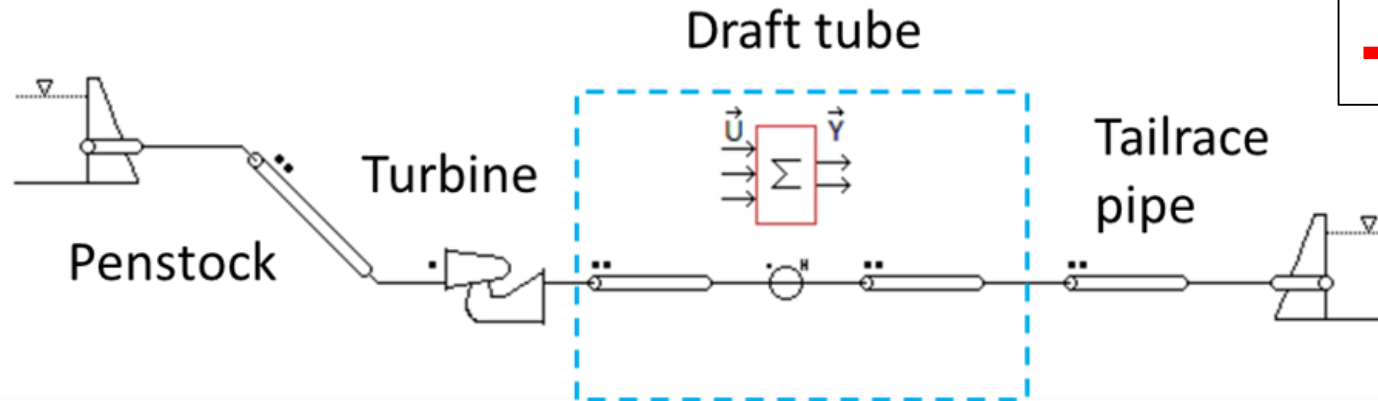
~~$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{l_{DT}}$$~~

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# Example of applications

- Hydraulic system 2



$$N_n = 750 \text{ rpm} = 12.5 \text{ Hz}$$

$$\rightarrow f_{\text{excitation}} = [0.2 - 0.4] \cdot f_n$$

$$\rightarrow f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$$

## Penstock

L	=	300	m
D	=	1.2	m
a	=	1'250	m/s
$\lambda$	=	0.012	-

## Turbine

P <sub>n</sub>	=	5	MW
Q <sub>n</sub>	=	5.0	m <sup>3</sup> /s
H <sub>n</sub>	=	100	mWC
N <sub>n</sub>	=	750	rpm
D <sub>ref</sub>	=	0.846	m
N <sub>q</sub>	=	53	-

## Draft tube

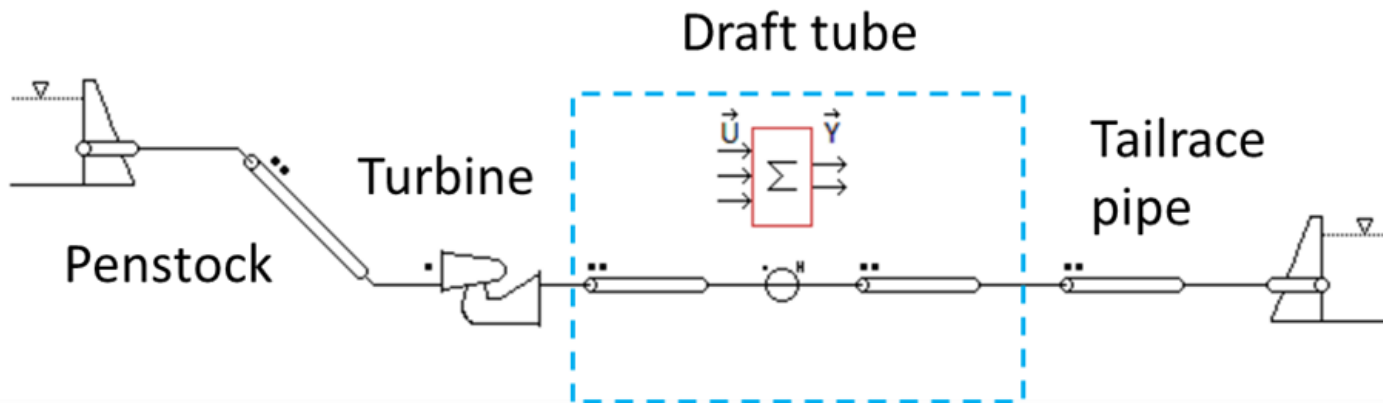
L	=	10	m
D	=	1.2	m
a	=	[50-100]	m/s
$\lambda$	=	0.012	-

## Tailrace pipe

L	=	100	m
D	=	1.2	m
a	=	1'250	m/s
$\lambda$	=	0.012	-

# Example of applications

## • Hydraulic system 2

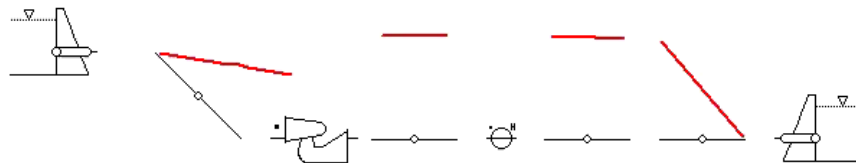


### Rigid column mode shape

- ✓ Linear amplitude variation of pressure in the TR
- ✓ Constant amplitude variation of discharge in the TR
- Similar to surge tank mass oscillation between TR pipe and DT compliance

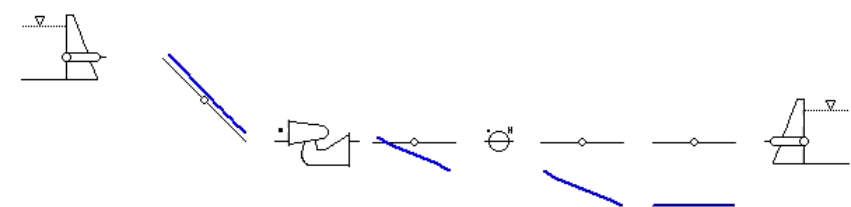
~~$$f_1 = \frac{a_{equ}}{\lambda_1} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot 1$$~~

$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \frac{A_{DT}}{A_{TR}}}}$$



1<sup>st</sup> Pressure mode

Rigid column mode shape

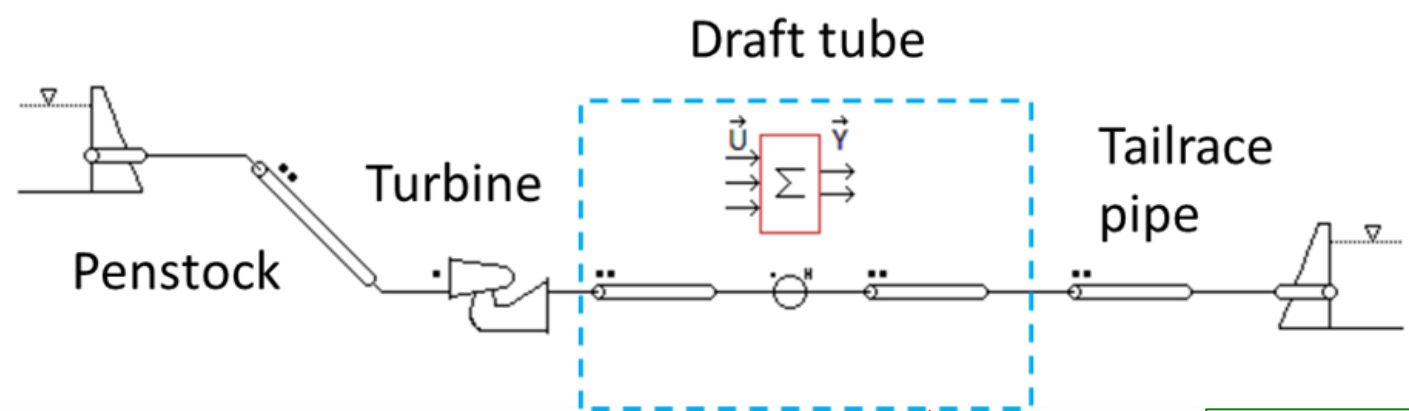


1<sup>st</sup> Discharge mode

# Example of applications

$f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$

- Hydraulic system 2



- 1<sup>st</sup> order: Good agreement with  $f_0$
- 2<sup>nd</sup>-6<sup>th</sup> order: Rather good agreement for natural frequencies  
Maximum error of 14%.
- Risk of resonance with the draft tube in red.

$$f_1 = \frac{a_{equ}}{\lambda_1} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot 1$$

$$f_0 = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \frac{A_{DT}}{A_{TR}}}}$$

$a = 50 \text{ m/s}$

$a = 100 \text{ m/s}$

System 2	Analytical calculation	Eigen value calculation	
	a DT (min)	a DT (min)	Relative error
	[m/s]	[m/s]	[%]
f0 [Hz]	0.25	0.27	-7.41

f1 [Hz]	0.96	0.27	255.56
f2 [Hz]	1.92	2.04	-5.88
f3 [Hz]	2.88	2.53	13.83
f4 [Hz]	3.85	4.12	-6.55
f5 [Hz]	4.81	4.87	-1.23
f6 [Hz]	5.77	5.16	11.82

System 2	Analytical calculation	Eigen value calculation	
	a DT (max)	a DT (max)	Relative error
	[m/s]	[m/s]	[%]
f0 [Hz]	0.5	0.55	-9.09

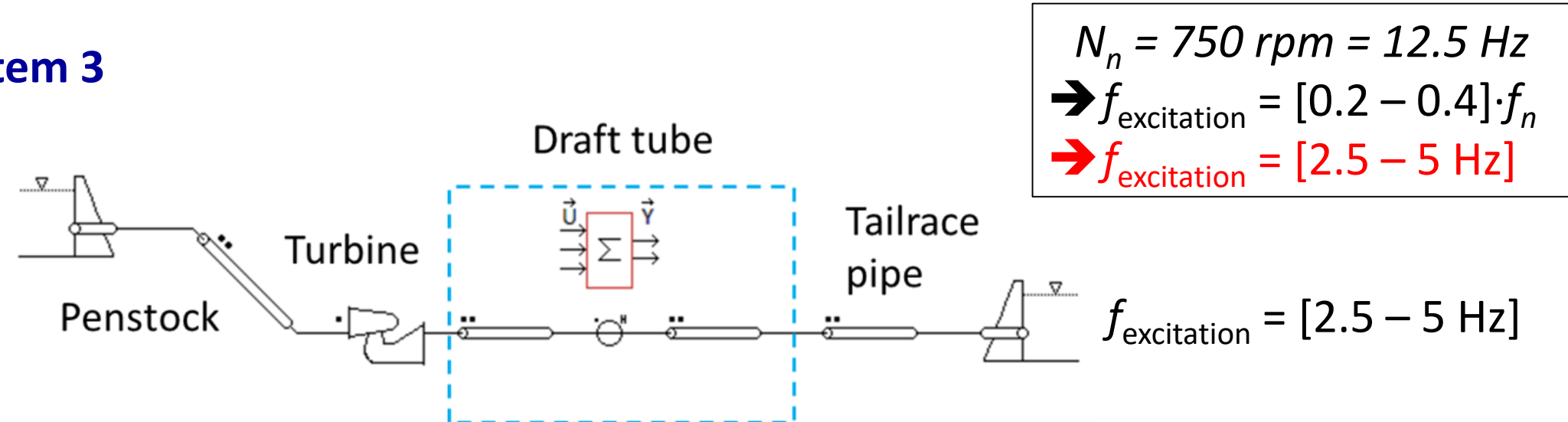
f1 [Hz]	1.19	0.55	116.36
f2 [Hz]	2.38	2.1	13.33
f3 [Hz]	3.57	4.05	-11.85
f4 [Hz]	4.76	4.88	-2.46
f5 [Hz]	5.95	6.18	-3.72
f6 [Hz]	7.14	6.36	12.26

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# Example of applications

- Hydraulic system 3



Penstock		
L	= 300	m
D	= 1.2	m
a	= 1'250	m/s
λ	= 0.012	-

Turbine		
P <sub>n</sub>	= 5	MW
Q <sub>n</sub>	= 5.0	m <sup>3</sup> /s
H <sub>n</sub>	= 100	mWC
N <sub>n</sub>	= 750	rpm
D <sub>ref</sub>	= 0.846	m
N <sub>q</sub>	= 53	-

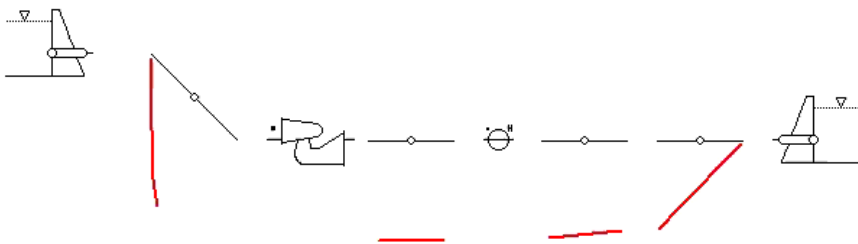
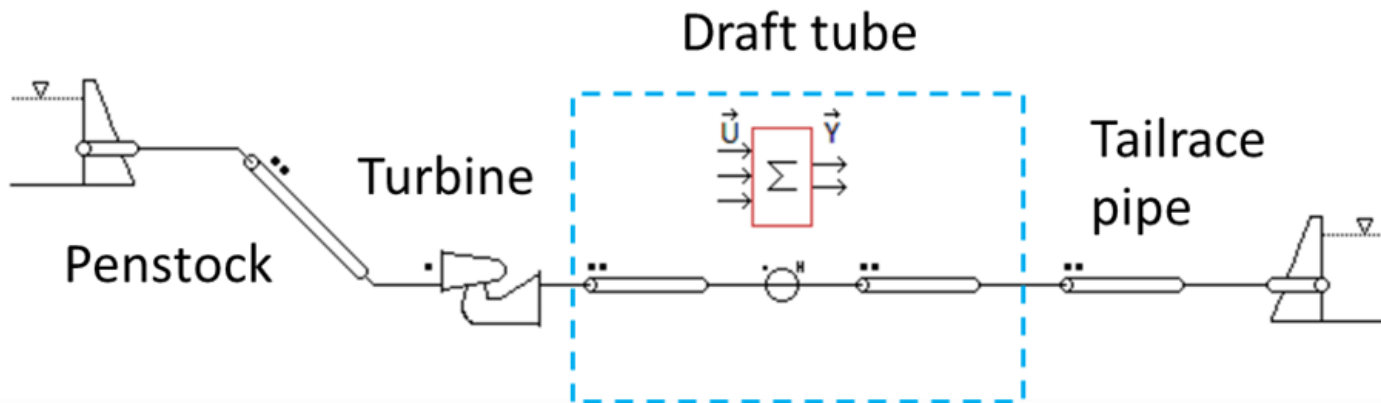
Draft tube		
L	= 10	m
D	= 1.2	m
a	= [50-100]	m/s
λ	= 0.012	-

Tailrace pipe		
L	= 100	m
D	= 2	m
a	= 1'250	m/s
λ	= 0.012	-

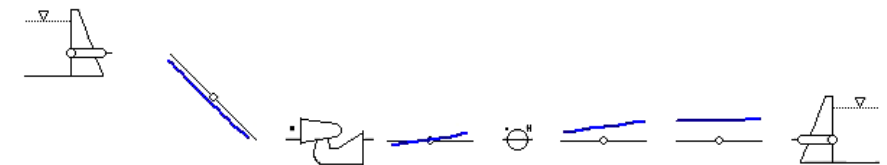


# Example of applications

## • Hydraulic system 3



1<sup>st</sup> Pressure mode



Rigid column mode shape

Rigid column mode shape

- ✓ Linear amplitude variation of pressure in the TR
- ✓ Constant amplitude variation of discharge in the TR
- Similar to surge tank mass oscillation between TR pipe and DT compliance

~~$$f_1 = \frac{a_{equ}}{\lambda_1} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot 1$$~~

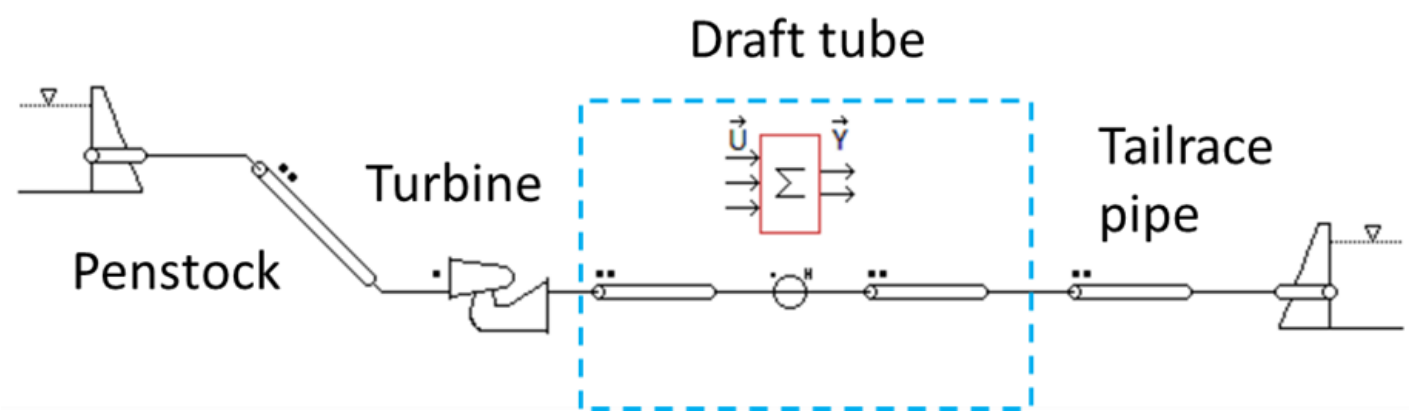
$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \frac{A_{DT}}{A_{TR}}}}$$

1<sup>st</sup> Discharge mode

# Example of applications

$f_{\text{excitation}} = [2.5 - 5 \text{ Hz}]$

- Hydraulic system 3



- 1<sup>st</sup> order: Good agreement with  $f_0$
- 2<sup>nd</sup>-6<sup>th</sup> order: Rather good agreement for natural frequencies  
Maximum error of 14%.
- Risk of resonance with the draft tube in red.

$a = 50 \text{ m/s}$

System 3	Analytical calculation	Eigen value calculation	Relative error
	a DT (min)	a DT (min)	
	[m/s]	[m/s]	
$f_0$ [Hz]	0.42	0.42	0.00
$f_1$ [Hz]	0.96	0.42	128.57
$f_2$ [Hz]	1.92	2.04	-5.88
$f_3$ [Hz]	2.88	2.55	12.94
$f_4$ [Hz]	3.85	4.12	-6.55
$f_5$ [Hz]	4.81	4.84	-0.62
$f_6$ [Hz]	5.77	6.16	-6.33

$a = 100 \text{ m/s}$

System 3	Analytical calculation	Eigen value calculation	Relative error
	a DT (max)	a DT (max)	
	[m/s]	[m/s]	
$f_0$ [Hz]	0.84	0.81	3.70
$f_1$ [Hz]	1.19	0.81	46.91
$f_2$ [Hz]	2.38	2.1	13.33
$f_3$ [Hz]	3.57	4.03	-11.41
$f_4$ [Hz]	4.76	4.72	0.85
$f_5$ [Hz]	5.95	6.14	-3.09
$f_6$ [Hz]	7.14	6.55	9.01

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## Conclusions

- Analytical approach to determine the possible risk of resonance with the draft tube vortex rope excitation (2.5-5Hz)
- **1<sup>st</sup> order:** Better agreement with  $f_1$  for the hydraulic system **without a tailrace pipe** (hydraulic system 1) → Elastic mode shape
- **1<sup>st</sup> order:** Good agreement with  $f_0$  for the hydraulic system **with a tailrace pipe** (hydraulic system 2 & 3) → rigid column mode shape
- **2<sup>nd</sup>-6<sup>th</sup> order:** Rather **good agreement for natural frequencies** (Maximum error of 14%).
- Limitations of the methodology
  - ✓ **Parallel branches:**
    - Modelling by a single branch with equivalent parameters to obtain a first order of magnitude.
    - Real system will feature much more complex and numerous eigenvalues (hydraulic system asymmetry, diameters, bifurcations)

# Take away Message

- This analytical method is included as ANNEXE E.2 of the new IEC Technical Specification 62882 ED1 (to be issued in 2020)

**Without a tailrace pipe**

**1<sup>st</sup> order**

$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$$

**2<sup>nd</sup>–6<sup>th</sup> order**

$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$$

**With a tailrace pipe**

**1<sup>st</sup> order**

$$f_o = \frac{1}{2 \cdot \pi} \frac{a_{DT}}{\sqrt{l_{DT} \cdot l_{TR} \frac{A_{DT}}{A_{TR}}}}$$

**2<sup>nd</sup>–6<sup>th</sup> order**

$$f_k = \frac{a_{equ}}{\lambda_k} = \frac{a_{equ}}{2 \cdot l_{tot}} \cdot k$$

Hydraulic machines – IEC Technical specification for Francis turbine pressure fluctuation

		<b>4/375/DTS</b>	
DRAFT TECHNICAL SPECIFICATION (DTS)			
PROJECT NUMBER: IEC TS 62882 ED1			
DATE OF CIRCULATION: 2019-08-30		CLOSING DATE FOR VOTING: 2019-11-22	
SUPERSEDES DOCUMENTS: 4/352/CD, 4/369A/CC			
IEC TC 4 : HYDRAULIC TURBINES			
SECRETARIAT: Canada		SECRETARY: Mr Robert Arseneault	
OF INTEREST TO THE FOLLOWING COMMITTEES: TC 2, TC 114			
FUNCTIONS CONCERNED: <input type="checkbox"/> EMC <input type="checkbox"/> ENVIRONMENT <input type="checkbox"/> QUALITY ASSURANCE <input type="checkbox"/> SAFETY			
This document is still under study and subject to change. It should not be used for reference purposes. Recipients of this document are invited to submit, with their comments, notification of any relevant patent rights of which they are aware and to provide supporting documentation.			
TITLE: Hydraulic machines – Technical specification for francis turbine pressure fluctuation transposition			
proposed stability date: 2022			
NOTE FROM TC/SC OFFICERS:			

# Thank you for your attention!

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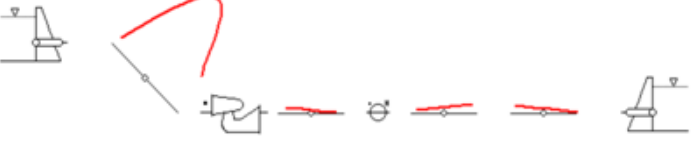
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# Example of applications

	Hydraulic system 1	Hydraulic system 2
<b>f<sub>1</sub></b>		
<b>f<sub>2</sub></b>		
<b>f<sub>3</sub></b>	